



Національний технічний університет України  
«КИЇВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ  
імені ІГОРЯ СІКОРСЬКОГО»



Ecology and technology of  
plant polymers

### **Basics of metrology and theory of errors**

#### **Working program of the academic discipline (Syllabus)**

##### **Details of the academic discipline**

<b>Level of higher education</b>	<i>Second (master's)</i>
<b>Branch of knowledge</b>	<i>16 Chemical and bioengineering</i>
<b>Specialty</b>	<i>161 Chemical technologies and engineering</i>
<b>Educational program</b>	<i>Industrial ecology and resource-efficient clean technologies</i>
<b>Discipline status</b>	<i>Selective</i>
<b>Form of education</b>	<i>correspondence (day)/distance/mixed</i>
<b>Year of training, semester</b>	<i>1st year, spring semester</i>
<b>Scope of the discipline</b>	<i>5.0 credits (150 hours)</i>
<b>Semester control/ control measures</b>	<i>Exam</i>
<b>Lessons schedule</b>	<i>26 hours (20 hours of lectures + 6 hours of practical classes)</i>
<b>Language of teaching</b>	<i>Ukrainian</i>
<b>Information about the course leader / teachers</b>	Lecturer: <a href="https://eco-paper.kpi.ua/pro-kafedru/vykladachi/vizytky/Ploskonos-Victor-Grigorovych.html">https://eco-paper.kpi.ua/pro-kafedru/vykladachi/vizytky/Ploskonos-Victor-Grigorovych.html</a> Practical / Seminar: <a href="https://eco-paper.kpi.ua/pro-kafedru/vykladachi/vizytky/Ploskonos-Victor-Grigorovych.html">https://eco-paper.kpi.ua/pro-kafedru/vykladachi/vizytky/Ploskonos-Victor-Grigorovych.html</a>
<b>Placement of the course</b>	<a href="https://do.ipk.kpi.ua/course/view.php?id=4395">https://do.ipk.kpi.ua/course/view.php?id=4395</a>

##### **Program of educational discipline**

#### **1. Description of the educational discipline, its purpose, subject of study and learning outcomes**

*The knowledge acquired in the process of studying the academic discipline provides an opportunity to acquire the skills of the basics of metrology and support in the appropriate condition of the measuring equipment (MT) for the proper metrological support of the technological processes of plant raw materials processing. They will make it possible to evaluate with the help of models the state of complex technological systems of paper and cardboard production, to explain the nature of errors that arise in the process of conducting experiments, to apply error theory methods for the analysis of random phenomena; identify and use a confidence interval to predict the state of systems; calculate the minimum possible errors of experimental research from laboratory and industrial conditions.*

*Subject of the educational discipline "Fundamentals of Metrology and Theory of Errors" are measured physical quantities, principles and methods of measuring physical quantities; basic concepts of probability theory and statistics; measuring equipment, their classification and purpose; metrology as a scientific basis for the unity and accuracy of measurements; metrological attestation of FTA: methods of verification (calibration) of FTA; estimation of errors in the case of multiple direct measurements; relative errors; formulas for calculating the sum, difference, product and share of measured values in case of independent errors; errors for using a satisfactory function of one variable; estimation of error characteristics and measurement uncertainty calculation; comparative analysis of two approaches to determining the characteristics of measurement accuracy; classification of complex technological*

systems; system analysis; procedures, methods and techniques of system analysis; identification of the characteristics of complex systems using the principles of self-organization and the topological method of analysis. To a large extent, the solution of the set tasks will be determined by the level of training of specialists who solve the issue of resource conservation, including scientific institutions and organizations, enterprises.

In order to successfully solve tasks, specialists must be fluent in information, able to solve complex problems of modeling situations at the highest scientific level.

### **The purpose of the educational discipline "Fundamentals of metrology and theory of errors"**

The goal of the educational discipline is the formation of students' competencies:

–Ability to organize work related to environmental assessment, environmental protection and optimization of nature use;

- The ability to assess the state of complex cardboard and paper production systems using mathematical models, draw conclusions about the reliability of such assessments and provide an engineering assessment of the accuracy of measurements using the HRT;

- The ability to use a confidence interval to forecast the state of cardboard and paper production systems at the stages of their development and operation.

1.2. According to the requirements of the program of the educational discipline "Fundamentals of Metrology and Theory of Errors", after mastering it, students must demonstrate the following learning outcomes:

- To know modern approaches to the organization of ecologically clean productions, reorganization and reconstruction of existing productions from the standpoint of resource conservation;

- Be able to analyze, argue, make decisions based on the analysis of complex technological systems for the production of cardboard and paper products using computer technologies, the basic provisions of the theory of errors, the regularities of the normal law of the distribution of random variables and the corresponding mathematical apparatus of the theory of errors; - Be able to choose the conditions for conducting experimental research in the laboratory (production) with minimal possible errors; justify the confidence interval of the processes under investigation in laboratory and production conditions.

## **2. Pre-requisites and post-requisites of the discipline (place in the structural and logical scheme of training according to the relevant educational program)**

Studying the discipline "Fundamentals of Metrology and Theory of Errors" is based on the principles of integration of various knowledge acquired by students during the bachelor's and the 1st semester of master's studies during the study of engineering disciplines. The discipline "Fundamentals of Metrology and Theory of Errors" is the foundation that should provide solution of technical problems and aimed at deep rethinking of existing and creation of new holistic knowledge and professional practice.

## **3. Content of the academic discipline**

### **SECTION 1. METROLOGY**

#### **Topic 1 The essence of metrology. Tasks and functions of metrology**

Systems of physical quantities and their units. Principles and methods of measuring physical quantities.

#### **Topic 2 Measuring equipment**

Means of measuring equipment, classification of measuring equipment and their purpose. Structure and parameters of measuring equipment. Accuracy indicators and forms of presentation of measurement results.

#### **Topic 3 State metrological service of Ukraine**

Structure and functions of metrological service of Ukraine. Law on metrology: basic concepts and

definitions.

**Topic 4 Metrological assurance of unity and accuracy of measurements**

Metrological assurance of unity and accuracy of measurements. State metrological control and supervision. State metrological service of the enterprise. Metrology as a scientific basis of unity and accuracy of measurements. State metrological control and supervision.

**Topic 5 Metrological attestation, verification and calibration of measuring equipment**

Metrological attestation, verification and calibration of measuring equipment. Methods of verification of FTAs. Metrological certification of measuring equipment. Verification methods (calibration). Verification schemes. Metrological verification of measuring equipment. Types of metrological checks. Organization and procedure of metrological verification.

**Topic 6 International and regional metrology organizations**

International metrology organizations. International Organization of Weights and Measures. International Organization of Legislative Metrology. Regional metrology organizations. Metrology in the countries of Western Europe. Metrology in the USA. Metrology in some countries of Eastern and Central Europe.

Metrological examination of technical documentation. General provisions and tasks of metrological examination. Organization and procedure of metrological examination. Metrological examination of design documentation. Metrological examination of technological documentation.

**CHAPTER 2. FUNDAMENTALS OF ERROR THEORY**

**Topic 1 Measurable physical quantities. Principles and methods of measuring physical quantities**

Measured physical quantities. Principles and methods of measuring physical quantities. Basic concepts of probability theory and statistics.

**Topic 2 Preliminary acquaintance with the accuracy of measurements**

Mistakes are like mistakes. The inevitability of errors. How important it is to know mistakes. Estimation of the error when counting from the scale. Estimation of errors in the case of multiple direct measurements.

**Topic 3 Basic provisions for ensuring the accuracy of measurements**

Best estimate  $\pm$  deviation. Significant numbers with defined deviations. The difference between the measurement results. Comparison of two values: measured and theoretically known. Comparison of two measured values. Multiplication of two measured values.

**Topic 4 Statistical analysis of multiple measurements with random deviations**

The sum and difference of the measured values. Multiplication and division of measured quantities. Multiplication of the measured value by an exact number. Raising the measured value to the power. Generalizing formulas for calculating the sum, difference, product and fraction of measured values. Accuracy of measurements using a satisfactory function of one variable. Step-by-step accuracy calculation method. The general formula for calculating the accuracy of measurements in indirect measurements.

**Topic 5 Statistical analysis of multiple measurements**

Random and systematic errors. Average value and standard deviation. Standard deviation is like the error of a unit measurement. Standard deviation of the mean. Systematic errors for experimental research in educational laboratories.

**Topic 6 Normal distribution of a statistical value**

Histograms and distribution of random variables. Limit distribution of random variables. The normal distribution of a random variable.

**Topic 7 Justification of error calculation formulas based on the law of normal distribution**

Standard deviation as 68% confidence interval. Justification of the mean as the best estimate and - the width of the marginal distribution.

**Topic 8 Calculation and justification of the confidence interval**

Calculation and justification of the confidence interval.

### **Topic 9 The problem of screening and combining measurement results**

The problem of sifting data. Chauvet's criterion. The problem of combining the results of different measurements.

### **Topic 10 Criterion $\chi^2$ for marginal distributions**

The concept of the  $\chi^2$  criterion. Degrees of freedom and reduced value of  $\chi^2$ . Probabilities for  $\chi^2$ .

### **Topic 11 Methods of assessing the accuracy of measurements based on the concept of uncertainty**

General concepts and definitions of the concept of uncertainty. An example of estimating error characteristics and calculating measurement uncertainty. Comparative analysis of two approaches to determining the characteristics of measurement accuracy.

### **Topic 12 Modeling of complex technological systems**

The concept of modeling complex technological systems in engineering and scientific activities. Object of study. Types of optimization parameters and requirements for them. Model selection. Regression analysis is one of the process modeling methods. Basic terms and concepts of regression analysis. The method of least squares. Compilation of a system of normal equations (multiple regression). Statistical treatment of regression equations. Correlation analysis. Multiple linear correlation.

### **Topic 13 Principles of mathematical self-organization of complex technological systems**

Principles of mathematical self-organization of complex technological systems. Method of group consideration of arguments (MGUA). Criteria of maximum informativeness and noise resistance of the experiment. Methods of processing the results of experimental studies.

## **4. Educational materials and resources**

### **Basic literature**

1. Nesterchuk D.M., Kvitka S.O., Halko S.V.. *Basics of metrology and measuring tools: a study guide / – Melitopol: Publishing and printing center "Lux", 2017. - 256 p.*
2. Bozhenko L.I. *Metrology, standardization, certification and accreditation. – Lviv: Afisha, 2016. - 324 p.*
3. Vasilevskiy O.M., Kucheruk V.Yu., Volodarskiy E.T. *Basics of the theory of measurement uncertainty: Textbook / – Vinnytsia: VNTU, 2015. – 230 p.*
4. Yermilova N.V., Kyslytsia S.G. *"New sources of standardization and methodology" : Study guide / – Poltava: PoltNTU, 2017. - 141 p.*

### **Additional literature**

5. *Law of Ukraine on metrology and metrological activity, No. 1765, Kyiv, 15.06. 2004*
6. *DSTU 3410-96 UkrSEPRO certification system. Substantive provisions.*
7. *Volodarskiy Y.T., Kuharchuk V.V., Podzharenko V.O., Serdyuk G.B. Metrological support of measurements and control. Study guide for technical students. university - Vinnytsia: Published. State, Technical University, 2011.-220 p.*
8. *DSTU 3651.2-97 Metrology. Units of physical quantities. Basic units of physical quantities. International systems of values. Basic provisions, names and designations.*
10. *Primakov SP., Barbash V.A. Technology of paper and cardboard. K.: ECMO, 2002.-396 p.*
11. *DSTU 2926-94 Quality systems. Complexes of quality management are system and technological. Substantive provisions.*

### **Information resources on the Internet**

Electronic resources from the course "Fundamentals of metrology and theory of errors", namely:

- credit module syllabus,
- methodical instructions for performing laboratory practicals and performing independent work

located at <http://www.eco-paper.kpi.ua/for-student>, as well as in the e-campus

Association of Ukrainian pulp and paper enterprises "UkrPapir" - [ukrbim@naverex.kiev.ua](mailto:ukrbim@naverex.kiev.ua)

### Educational content

#### 5. Methods of mastering an educational discipline (educational component)

##### Lecture classes

Lectures are aimed at:

- provision of modern, integral, interdependent knowledge in the discipline "Fundamentals of Metrology and Theory of Errors", the level of which is determined by the target attitude to each specific topic;
- ensuring creative work of students together with the teacher during the lecture;
- education of students' professional and business qualities and development of their independent creative thinking;
- forming the necessary interest in students and providing direction for independent work;
- definition at the current level of scientific development in the field of metrology and measurement accuracy;
- reflection of the methodical processing of the material (highlighting of the main provisions, conclusions, recommendations, their wording is clear and adequate);
- the use of visual materials for demonstration, combining them, if possible, with the demonstration of research results;
- teaching research materials in clear and high-quality language with observance of structural and logical connections, clarification of all newly introduced terms and concepts;
- accessibility for perception by this audience.

No. z/p	<i>The name of the topic of the lecture and a list of the main questions (a list of didactic tools, references to the literature and tasks on the SRS)</i>	Hour
1	<p style="text-align: center;"><b>SECTION I METROLOGY</b></p> <p><b>Topic 1 The essence of metrological assurance of product conformity assessment. Tasks and functions of metrological assurance of product conformity assessment.</b></p> <p>Lecture No. 1. Tasks and functions of metrology. Law on metrology: basic concepts and definitions.</p> <p>Literature: [1] p.10-23; [2] p.6-14, [3] p.7-19, [4] p.128-134.</p> <p>Tasks on SRS Principles and methods of measuring physical quantities.</p>	1
2	<p><b>Topic 2. Means of measuring technology (MET), classification of MT and their purpose</b></p> <p>Lecture No. 2. Measuring equipment, their classification and purpose. Structure and parameters of measuring equipment. Literature: [1] pp. 30-43; [2] pp. 16-34, [4] pp. 137-149, [14] pp. 28-44.</p> <p>Tasks on SRS Accuracy indicators and forms of presentation of measurement results.</p>	1
3	<p><b>Topic 3 State metrological service of Ukraine</b></p> <p>Lecture No. 3. Structure and functions of metrological service of Ukraine.</p> <p>Literature: [1] pp. 48-63; [2] pp. 36-44, [4] pp. 152-164, [14] pp. 48-64.</p> <p>Tasks on SRS Law on metrology: basic concepts and definitions.</p>	1

4	<p><b>Topic 4 Metrological assurance of unity and accuracy of measurements. State metrological control and supervision. Metrological service of the enterprise</b></p> <p>Lecture No. 4. Metrology as a scientific basis for the unity and accuracy of measurements. State metrological control and supervision.</p> <p>Literature: [2] p.48-66; [3] pp. 56-74, [4] pp. 166-184, [14] pp. 68-86.</p> <p>Tasks on SRS State metrological service of the enterprise (organization).</p>	1
5	<p><b>Topic 5 Metrological attestation, verification and calibration of measuring equipment. Methods of verification of FTAs</b></p> <p>Lecture No. 5. Metrological certification of measuring equipment. Verification methods (calibration). Verification schemes.</p> <p>Literature: [2] p.68-96; [3] pp. 76-94, [4] pp. 186-199, [14] pp. 88-106.</p> <p>Tasks on SRS Metrological verification of measuring equipment. Types of metrological checks. Organization and procedure of metrological verification.</p>	1
6	<p><b>Topic 6 International metrology organizations. Metrology in foreign countries</b></p> <p>Lectures No. 6-7. International Organization for Metrology. International Organization of Weights and Measures. International Organization of Legislative Metrology. Regional metrology organizations. Metrology in the countries of Western Europe. Metrology in the USA. Metrology in some countries of Eastern and Central Europe.</p> <p>Literature: [2] p.98-126; [3] pp. 96-124, [4] pp. 202-249, [14] pp. 108-136.</p> <p>Tasks on SRS Metrological examination of technical documentation. General provisions and tasks of metrological examination. Organization and procedure of metrological examination. Metrological examination of design documentation. Metrological examination of technological documentation.</p>	1
7	<p><b>CHAPTER 2. FUNDAMENTALS OF ERROR THEORY</b></p> <p><b>Topic 1 Measurable physical quantities. Principles and methods of measuring physical quantities</b></p> <p>Lecture No. 1. Measurable physical quantities. Principles and methods of measuring physical quantities. Basic concepts of probability theory and statistics.</p> <p>Literature: [5] pp. 12-19; [7] pp. 6-14; [8] pp. 5-17.</p> <p>Tasks on SRS. Systems of physical quantities and their units. Accuracy indicators and forms of presentation of measurement results. General provisions and classification of errors.</p>	1
8	<p><b>Topic 2 Preliminary acquaintance with the accuracy of measurements</b></p> <p>Lecture No. 2. Mistakes are like mistakes. The inevitability of errors. How important it is to know mistakes. Estimation of the error when counting from the scale. Estimation of errors in the case of multiple direct measurements.</p> <p>Literature: [5] pp. 21-29; [7] pp. 16-21; [8] pp. 25-37.</p> <p>Tasks on SRS. General provisions and classification of errors.</p>	1
9	<p><b>Topic 3 Basic provisions of measurement accuracy</b></p> <p>Lecture No. 3. Best estimate <math>\pm</math> accuracy. The difference between the measurement results. Comparison of two values: measured and theoretically known. Comparison of two measured values.</p> <p>Lecture No. 4. Comparison of two values: measured and theoretically known. Comparison of two measured values.</p> <p>Lecture No. 5. Relative errors. Significant figures are in relative errors. Multiplication of two measured values</p> <p>Literature: [5] p.31-57; [7] pp. 23-54; [8] pp. 39-68.</p>	1

	<i>Tasks on SRS. Comparison of two values: measured and theoretically known. Comparison of two measured values.</i>	
10	<p><b>Topic 4 Accuracy in indirect measurements</b></p> <p>Lecture No. 6. Errors of the sum and difference of measured quantities. Multiplication and division of measured quantities. Multiplication of the measured value by an exact number. Raising the measured value to the power.</p> <p>Lecture No. 7. Independent errors when calculating the sum of measured values. Generalizing formulas for calculating the sum, difference, product, and fraction of measured values with independent errors.</p> <p>Lecture No. 8. Errors when using a satisfactory function of one variable. Step-by-step error calculation method. General formula for calculating errors in indirect measurements.</p> <p>Literature: [5] p.61-77; [7] pp. 63-85; [8] pp. 69-88.</p> <p>Tasks on SRS. General information about errors in indirect measurements. Understanding errors in indirect measurements using examples. The principle of the arithmetic mean.</p>	2
11	<p><b>Topic 5 Statistical analysis of multiple measurements with random errors</b></p> <p>Lecture No. 9. Random and systematic errors. Average value and standard deviation.</p> <p>Lecture No. 10. Standard deviation is the error of a unit measurement. Standard deviation of the mean. Systematic errors for experimental research in educational laboratories.</p> <p>Literature: [5] c.106-130, [10] c.54-72, [6] c. 88-101.</p> <p>Tasks on SRS. Determination of the guarantee interval of measurement results. Summary of measurement errors. Errors of direct equal-precision measurements. Processing and assessment of the accuracy of exact measurements. Standard deviation of the mean in examples.</p>	1
12	<p><b>Topic 6 Normal distribution of statistical value</b></p> <p>Lecture No. 11. Histograms and distribution of random variables. Limit distribution of random variables. The normal distribution of a random variable.</p> <p>Literature: [5] c.136-155, [3] c.54-72, [10] c. 88-101.</p> <p>Tasks on SRS. The law of probability distribution for multiple measurements. Random variables. The use of elements of the theory of probabilities to the results of measurements. Repetition of tests - binomial distribution.</p>	1
13	<p><b>Topic 7 Justification of error calculation formulas based on the law of normal distribution</b></p> <p>Lecture No. 12. Standard deviation as a 68% confidence interval. Justification of the average <math>\bar{x}</math> as the best estimate and <math>\sigma</math> - the width of the marginal distribution.</p> <p>Lecture No. 13. Justification of calculation of errors in indirect measurements. Justification of the standard deviation of the mean.</p> <p>Literature: [5] c. 148-172; [7] c. 78-95.</p> <p>Tasks on SRS. Quadratic sum of errors and its justification. General case. Determination of the mean squared error.</p>	1
14	<p><b>Topic 8 Calculation and justification of the confidence interval</b></p> <p>Lecture No. 14. Calculation and justification of the confidence interval.</p> <p>Literature: [6] c. 123-155; [7] c. 108-123.</p> <p>Tasks on SRS. Confidence intervals. The required number of random variable measurements.</p>	1
15	<p><b>Topic 9 The problem of screening and combining measurement results</b></p> <p>Lecture No. 15. The problem of data screening. Chauvet's criterion. The problem of combining the results of different measurements.</p> <p>Literature: [5] c. 178-192; [13] c. 98-137.</p> <p>Tasks on SRS. The problem of data screening using the Chauvenet criterion on examples. The problem of combining the results of various measurements on examples.</p>	1
16	<b>Topic 10 Criterion <math>\chi^2</math> for marginal distributions</b>	1

	<p>Lecture No. 16. Concept of the <math>\chi^2</math> criterion. Degrees of freedom and reduced value of <math>\chi^2</math>. Probabilities for <math>\chi^2</math>.</p> <p>Lecture No. 17. An example of the development of a typical method of performing measurements to determine the mass fraction of kaolin in an aqueous suspension.</p> <p>Literature: [5] c. 198-222; [7] c. 218-141.</p> <p>Tasks on SRS. Linear correlation coefficient. quantitative criterion of significance <math>r</math>.</p>	
17	<p><b>Topic 11 Methods of assessing the accuracy of measurements based on the concept of uncertainty</b></p> <p>Lecture No. 18. General concepts and definition of the concept of uncertainty. An example of estimating error characteristics and calculating measurement uncertainty. Comparative analysis of two approaches to determining the characteristics of measurement accuracy.</p> <p>Literature: [6] c. 168-182.</p> <p>Tasks on SRS. Comparative analysis of two approaches to determining the characteristics of measurement accuracy.</p>	1
18	<p><b>Topic 12 Modeling of complex technological systems</b></p> <p>Lecture No. 19. The concept of modeling complex technological systems in engineering and scientific activities. Object of study. Types of optimization parameters and requirements for them. Model selection. Regression analysis is one of the process modeling methods. Basic terms and concepts of regression analysis. The method of least squares. Compilation of a system of normal equations (multiple regression). Statistical treatment of regression equations. Correlation analysis.</p> <p>Literature: [4] c. 65-82.</p> <p>Tasks on SRS. Multiple linear correlation.</p>	1
19	<p><b>Topic 13 Principles of mathematical self-organization of complex technological systems</b></p> <p>Lecture No. 20. Principles of mathematical self-organization of complex technological systems. Method of group consideration of arguments (MGUA). Criteria of maximum informativeness and noise resistance of the experiment.</p> <p>Literature: [4] c. 88-102.</p> <p>Tasks on SRS. Methods of processing the results of experimental studies.</p>	1
	<b>In total</b>	<b>20</b>

## Practical training

In the system of professional training of students in this discipline, practical classes occupy 15% of the classroom load. They lay and form the foundations of students' qualifications. The content of these classes and the method of conducting them should ensure the development of the creative activity of the individual. They develop scientific thinking and the ability to use special terminology, allow you to check knowledge, therefore this type of work is an important means of operational feedback. Practical classes should perform not only cognitive and educational functions, but also contribute to the growth of students as creative workers.

The main tasks of the cycle of practical classes and laboratory workshops:

- help students systematize, consolidate and deepen knowledge of a theoretical nature in the field of standardization, metrology and measurement accuracy;
- to teach their work with scientific and reference literature;
- to form skills to learn independently, that is, to master the methods, methods and techniques of self-learning, self-development and self-control.

No. z/p	Name of the subject of the practical session and list of main questions (a list of didactic support, references to the literature and tasks on the SRS)	Hour
<u>1</u>	<p><b>Practical lesson 1-2.</b></p> <p>General concepts of physical quantities. Systems of physical quantities. Clarification of the causes of errors. The main provisions of measurement accuracy. Solving problems for the purpose of general estimation of errors in the case of multiple measurements. Relative errors and significant figures.</p> <p>Literature: [5] p12-21; [7] pp. 6-14; [8] p. 16-23.</p>	1



	<i>Tasks on SRS. Solving problems for the purpose of general estimation of errors in the case of repeated measurements in laboratory conditions.</i>	
2	<b><u>Practical lesson 3-4.</u></b> <i>Errors in indirect measurements. Determining errors when using measurement results in sum, difference, multiplication and division operations Literature: [5] c. 49-64; [7] c.16-22.</i> <i>Tasks on SRS. Errors in indirect measurements in laboratory conditions.</i>	1
3	<b><u>Practical lesson 5-6.</u></b> <i>Statistical analysis of random errors. Calculation of mean and standard deviation. Literature: [5] c.87-91, [7] c.34-43, [8] c. 49-54.</i> <i>Tasks on SRS. Calculation of mean and standard deviation in laboratory conditions.</i>	1
4	<b><u>Practical lesson 7.</u></b> <i>Statistical analysis of random errors. Calculation of the standard deviation of the mean. Systematic errors. Literature: [5] c.92-101, [10] c.44-63, [13] c. 55-64.</i> <i>Tasks on SRS. Calculation of the standard deviation of the mean in laboratory conditions.</i>	1
5	<b><u>Practical lesson 8-9.</u></b> <i>Solving tasks for sifting and combining measurement results. Literature: [5] c. 148-152; [8] c. 18-58.</i> <i>Tasks on SRS. Screening and combining of measurement results in laboratory conditions in laboratory conditions.</i>	2
	<b><i>In total</i></b>	<b>6</b>

## 9. Independent work of the student

*Independent work takes up 52% of the credit module study time, including exam preparation. The main task of students' independent work is the mastery of scientific knowledge in areas that are not included in the list of theoretical foundations through personal search for information, formation of active interest in a creative approach in educational work. In the process of independent work within the framework of the educational component, the student must learn to analyze modern methods of assessing the accuracy of metrological measurements and developing mathematical models.*

No. z/p	The name of the topic submitted for independent processing	Number of hours of SRS
<b>Chapter 1 Metrology</b>		
1	<b>Topic 1 The essence of metrological assurance of product conformity assessment. Tasks and functions of metrological assurance of product conformity assessment.</b> <i>SRS to topic 1 Principles and methods of measuring physical quantities. Literature: [1] p.10-23; [2] p.6-14, [3] p.7-19, [4] p.128-134.</i> <b>Topic 2. Means of measuring technology (MET), classification of MT and their purpose</b> <i>SRS to topic 2 Accuracy indicators and forms of presentation of measurement results. Literature: [1] pp. 30-43; [2] pp. 16-34, [4] pp. 137-149, [14] pp. 28-44.</i> <b>Topic 3 State metrological service of Ukraine</b> <i>SRS to topic 3 Law on metrology: basic concepts and definitions.</i> <b>Topic 4 Metrological assurance of unity and accuracy of measurements. State metrological control and supervision. Metrological service of the enterprise</b>	40

	<p><i>SRS to topic 4 State metrological service of the enterprise (organization). Literature: [2] p.48-66; [3] pp. 56-74, [4] pp. 166-184, [14] pp. 68-86.</i></p> <p><b>Topic 5 Metrological attestation, verification and calibration of measuring equipment. Methods of verification of FTAs</b></p> <p><i>SRS to topic 5 Metrological verification of measuring equipment. Types of metrological checks. Organization and procedure of metrological verification. Literature: [2] p.68-96; [3] pp. 76-94, [4] pp. 186-199, [14] pp. 88-106.</i></p> <p><b>Topic 6 International metrology organizations. Metrology in foreign countries</b></p> <p><i>SRS to topic 6 Metrological examination of technical documentation. General provisions and tasks of metrological examination. Organization and procedure of metrological examination. Metrological examination of design documentation. Metrological examination of technological documentation. Literature: [2] p.98-126; [3] pp. 96-124, [4] pp. 202-249, [14] pp. 108-136.</i></p>	
<b>Chapter 2. Fundamentals of error theory</b>		
2	<p><b>Topic 1 Measurable physical quantities. Principles and methods of measuring physical quantities</b></p> <p><i>SRS to topic 1 Systems of physical quantities and their units. Accuracy indicators and forms of presentation of measurement results. General provisions and classification of errors. Literature: [5] pp. 12-19; [7] pp. 6-14; [8] pp. 5-17.</i></p> <p><b>Topic 2 Preliminary acquaintance with the accuracy of measurements</b></p> <p><i>SRS to topic 2 General provisions and classification of errors. Literature: [5] pp. 21-29; [7] pp. 16-21; [8] pp. 25-37.</i></p> <p><b>Topic 3 Basic provisions of measurement accuracy</b></p> <p><i>SRS to topic 3 Comparison of two values: measured and theoretically known. Comparison of two measured values. Literature: [5] p.31-57; [7] pp. 23-54; [8] pp. 39-68.</i></p> <p><b>Topic 4 Errors in indirect measurements</b></p> <p><i>SRS to topic 4 General information about errors in indirect measurements. Understanding errors in indirect measurements using examples. The principle of the arithmetic mean. Literature: [5] p.61-77; [7] pp. 63-85; [8] pp. 69-88.</i></p> <p><b>Topic 5 Statistical analysis of multiple measurements with random errors</b></p> <p><i>SRS to topic 5 Determination of the guarantee interval of measurement results. Summary of measurement errors. Errors of direct equal-precision measurements. Processing and assessment of the accuracy of exact measurements. Standard deviation of the mean in examples. Literature: [5] c.106-130, [10] c.54-72, [6] c. 88-101.</i></p> <p><b>Topic 6 Normal distribution of statistical value</b></p> <p><i>SRS to topic 6 The law of probability distribution for multiple measurements. Random variables. The use of elements of the theory of probabilities to the results of measurements. Repetition of tests - binomial distribution. Literature: [5] c.136-155, [3] c.54-72, [10] c. 88-101.</i></p> <p><b>Topic 7 Justification of error calculation formulas based on the law of normal distribution</b></p> <p><i>SRS to topic 7 Quadratic sum of errors and its justification. General case. Determination of the mean squared error. Literature: [5] c. 148-172; [7] c. 78-95.</i></p> <p><b>Topic 8 Calculation and justification of the confidence interval</b></p> <p><i>CRC to Topic 8 Confidence Intervals. The required number of random variable measurements. Literature: [6] c. 123-155; [7] c. 108-123.</i></p> <p><b>Topic 9 The problem of screening and combining measurement results</b></p> <p><i>SRS to topic 9 The problem of screening data using the Chauvenet criterion with</i></p>	50

	<p>examples. The problem of combining the results of various measurements on examples.  <i>Literature: [5] c. 178-192; [13] c. 98-137.</i></p> <p><b>Topic 10 Criterion <math>\chi^2</math> for marginal distributions</b>  <i>SRS to topic 10 Coefficient of linear correlation. quantitative criterion of significance <math>r</math>.</i>  <i>Literature: [5] c. 198-222; [7] c. 218-141.</i></p> <p><b>Topic 11 Methods of assessing the accuracy of measurements based on the concept of uncertainty</b>  <i>SRS to topic 11 Comparative analysis of two approaches to determining the characteristics of measurement accuracy.</i>  <i>Literature: [6] c. 168-182.</i></p>	
4	<p><b>Topic 12 Modeling of complex technological systems</b>  <i>SRS to topic 12 Multiple linear correlation.</i>  <i>Literature: [4] c. 65-82</i></p> <p><b>Topic 13 Principles of mathematical self-organization of complex technological systems</b>  <i>SRS to topic 13 Methods of processing the results of experimental studies.</i>  <i>Literature: [4] c. 108-123</i></p>	11
5	<b>Preparation for MKR</b>	3
6	Preparation for the exam	20
	Hours in general	124

## Policy and control

### 10. Policy of academic discipline (educational component)

#### Rules of attending classes and behavior in classes

Attending classes is a mandatory component of the assessment. Students are obliged to take an active part in the educational process, not to be late for classes and not to miss them without a good reason, not to interfere with the teacher conducting classes, not to be distracted by actions unrelated to the educational process.

#### Rules for assigning incentive and penalty points

- incentive points can be awarded by the teacher exclusively for the performance of creative works in the discipline or additional completion of online specialized courses with the receipt of the appropriate certificate:
- <https://www.coursera.org/learn/research-methods>;
- <https://ru.coursera.org/learn/metodologiya-nauchnyh-issledovanij-kotiki>.

But their sum cannot exceed 10% of the rating scale.

- Penalty points are not provided within the academic discipline.

#### Policy of deadlines and rescheduling

In the event of arrears from the academic discipline or any force majeure circumstances, students should contact the teacher through the available (provided by the teacher) communication channels to resolve problematic issues and agree on the algorithm of actions for practice.

## Policy of academic integrity

Plagiarism and other forms of dishonest work are unacceptable. Plagiarism includes the absence of references for the use of printed and electronic materials, quotes, opinions of other authors. Inadmissible tips and write-offs during writing tests, conducting classes; passing a test for another student; copying materials protected by the copyright system without the permission of the author of the work.

The policy and principles of academic integrity are defined in Chapter 3 of the Code of Honor of the National Technical University of Ukraine "Ihor Sikorsky Kyiv Polytechnic Institute". More details:<https://kpi.ua/code>

## Policy of academic behavior and ethics

Students should be tolerant, respect the opinions of others, formulate objections in the correct form, constructively support feedback during classes.

Standards of ethical behavior of students and employees are defined in Chapter 2 of the Code of Honor of the National Technical University of Ukraine "Ihor Sikorskyi Kyiv Polytechnic Institute". More details:<https://kpi.ua/code>

## 8. Types of control and rating system for evaluating learning outcomes (RSO)

Distribution of study time by types of classes and tasks in the discipline according to the working study plan:

Semester	Training time		Distribution of study hours				Control measures		
	Credits	Acad. hours	Lectures	Practical	Lab. practice	SRS	MKR	DKR	Semester control
3	5.0	150	20	6	-	124	1	-	exam

### The student's rating in the discipline consists of the points he receives for:

The rating from the credit module consists of points that he receives for work in practical classes and completion of MKR.

Semester control is an exam.

### System of rating (weighted) points and evaluation criteria

The system of rating points and evaluation criteria:

Performance of tasks in practical classes.

Thus, the weighted score of the answer in the practical lesson ( $rn$ ) = 4 points. The maximum number of points in all practical classes is equal to:

$$4 \text{ points} \times 9 = 36 \text{ points.}$$

Weight score MKR-1 and MKR-2 ( $rmkr$ ) = 7 points each. The maximum number of points for all test papers is equal to:  $7 \text{ points} \times 2 = 14 \text{ points}$ .

The maximum amount of points of the starting component during the semester is equal to:

$$(4 \times 9) + (2 \times 7) = 50 \text{ points}$$

Under the condition of good preparation and active work in a practical lesson (or laboratory workshop) - 1 point. One or two best students in each practical lesson can be given 1 point as an incentive.

*A prerequisite for admission to the exam is a starting rating of at least 23 points.*

*According to the results of educational work in the first 7 weeks, the "ideal student" should score 16 points. At the first certification (8th week), the student receives "passed" if his current rating is at least  $0.5 \times 16 = 8$  points.*

*According to the results of 13 weeks, the "ideal student" should score 32 points. At the second certification (14th week), the student receives "credited" if his current rating is at least  $0.5 \times 32 = 16$  points.*

**The size of the examination scale is RE = 50 points (50% of R)**

**Conditions for admission to the exam:***starting rating of at least 23 points ( $rc \geq 23$  points, at least 50% of RS).*

**Examination assessment criteria**

*The exam ticket contains one theoretical question that has a weighting point of 30 and one practical task that has a weighting point of 25. The maximum number of points is  $30 + 25 = 55$  points.*

**The evaluation system of theoretical questions:**

- *"excellent", complete answer (at least 90% of the required information) - 30-28 points;*
- *"good", sufficiently complete answer (at least 75% of the required information, or minor inaccuracies) - 23-22 points;*
- *"satisfactory", incomplete answer (at least 60% of the required information and some errors) - 18-16 points;*
- *"unsatisfactory", unsatisfactory answer - 0 points.*

**Assessment system of practical classes(or labs):**

- *"excellent", complete answer (at least 90% of the required information) - 23-22 points;*
- *"good", sufficiently complete answer (at least 75% of the required information, or minor inaccuracies) - 19-17 points;*
- *"satisfactory", incomplete answer (at least 60% of the required information and some errors) - 15-13 points;*
- *"unsatisfactory", unsatisfactory answer - 0 points.*

*The sum of starting points and points for answers to questions on the exam ticket is converted to the exam grade according to the table:*

<b>Scores</b>	<b>Rating</b>
95...100	perfectly
85...94	very good
75...84	fine
65...74	satisfactorily
60...64	enough
RD < 60	unsatisfactorily
Admission conditions not met	not allowed

**11. Additional information on the discipline (educational component)**

**List of questions of modular control works**

### Modular control work (mcr-1)

1. Define metrology as a science.
2. What are the main tasks of metrology?
3. Show what problems the tasks of metrology are aimed at solving?
4. In accordance with tasks and functions, what types of metrology can be called?
5. In accordance with the law "On metrology and metrological activities" define the unity of measurements.
6. Name the main organization for ensuring the unity of measurements in Ukraine.
7. What refers to departmental metrological services of Ukraine?
8. Define the measuring equipment.
9. List the types of measuring equipment?
10. Show why all FTAs are subject to mandatory state departmental verification?
11. What constitutes the basis of the state system of ensuring the unity of measurements?
12. Define what is called metrological support?
13. Bring at the expense of what is the unity of measurements achieved and to determine the unanimity of the FTA?
14. Show what constitutes the technical basis of metrological support?
15. What are the objects of State control and supervision?

### Modular control work (mcr-2)

1. Define the term "error theory".
2. Define what the difference in measurement results is? Significant/insignificant difference.
3. Define what a relative error is? Give the relative error formula.
4. Give the rule of error of the product and division of the results of indirect measurements.
5. Give the error rule of the product of the result of indirect measurements by an exact number.
6. Bring generalizing formulas for calculating the product and fraction of measured values for independent errors.
7. Give the formula for calculating the error in a satisfactory function of one variable.
8. Show by example how the confidence interval changes depending on the number of parallel experiments?
9. Define the problem of screening out abnormal results. Cite the Chauvin criterion?
10. Define the problem of combining the results of  $n$  measurements. Give the rule of association and formulas for calculation  $x_{i \text{ a e e d.}}$  and  $\omega_{x_{i \text{ a e e d.}}}$  ?

### An approximate list of questions that are submitted for semester control:

1. Justify the concept and provide a formula for calculating the relative error.
2. To justify the concept of measurement accuracy through relative error.
3. To justify the use of the concept of relative error in the formula for calculating the error of obtaining measurement results.
4. Analyze and provide a rule for calculating the error of the sum and difference of two independent measured values.
5. Analyze the error calculation rules used when measuring independent measured quantities.
6. Analyze and provide the rule for calculating the error of the sum and the difference of values, the rule for the error of the product and division of the measurement results, as well as the rule for the product of the measurement result by an exact number.
7. To justify the use of the rule for calculating the error when raising the measured value to the power.

8. Analyze and provide a formula for estimating the error when using a satisfactory function of one variable in cases of indirect measurements.
9. To justify the use of the general formula for calculating errors in indirect measurements and the step-by-step method.
10. To justify the essence of the problem of combining the results of the experiment and to decide on the formulas for calculating the weighted average.
11. Define what the difference between measurement results is, give an estimate of the significance (insignificance) of the difference, based on the concept of best estimate and error.
12. Analyze and provide a formula for calculating the standard deviation of the mean.
13. Analyze and provide a rule for calculating the error of the difference of measurement results.
14. Analyze and provide formulas for calculating the value of the  $\chi^2$  criterion.
15. To justify the essence of the problem of combining the results of the experiment and to decide on the formulas for calculating the weighted average.
16. Analyze and provide a sequence of formulas for calculating a confidence interval.
17. To substantiate the scheme of using the Chauvenet criterion.
18. To justify the essence of the problem of combining the results of the experiment and to decide on the formulas for calculating the weighted average.
19. Analyze and provide formulas for statistical processing of measurement results, namely: calculation of the average value and standard deviation of the average.
20. To analyze the error estimation formula for the use of a satisfactory function of one variable in cases of indirect measurements.
21. To substantiate the scheme of using the Chauvenet criterion.
22. Analyze and provide formulas and determine the sequence (algorithm) of using the formulas to prove whether a certain sample of observations corresponds to the Gaussian normal distribution.
23. Analyze and provide formulas for calculating the value of the  $\chi^2$  criterion.
24. Analyze and provide formulas for statistical processing of measurement results, namely: calculation of the average value and standard deviation of the average.

### **Tasks**

1. Apply the formula for calculating the relative error (in percent) for five measurements:
  - measured height =  $5.03 \pm 0.04$  m;
  - measured time =  $19.5 \pm 1$  s;
  - measured charge =  $(-3.2 \pm 0.3) \cdot 10^{-19}$  K);
  - measured wavelength =  $(0.56 \pm 0.07) \cdot 10^{-6}$  m);
  - measured impulse =  $(3.27 \pm 0.04) \cdot 10^3$  g\*cm/s).
  
2. Use the concept of measurement accuracy due to the relative error for the case, namely: suppose you need to measure a length of 2 cm with an accuracy of 1%. With the help of a wooden ruler, you can count with an accuracy of up to 1 mm, and with the help of a microscope - up to 0.1 mm.  
Is it possible to do this with a wooden ruler? With a microscope?
  
3. Use the formula for calculating the error of measurement results in the case when two values  $a$  and  $b$  are measured (the length and width of the paper strip for strength testing). We get:  $a = 11.5 \pm 0.2$  cm and  $c = 25.4 \pm 0.2$  cm.  
It is necessary to calculate the value of the area of the strip  $S = a \cdot b$ , the absolute and relative value of the error in  $S$ , as well as the relative value of the errors of the measured values?

4. Use the rule for calculating the error of the sum and the difference of two independent measured values in the case of:

The laboratory assistant mixes solutions of chemical reagents from two bottles, having previously measured separately the masses of these filled and then empty bottles and obtained as a result:

$M_1$  - mass of the first cylinder and its contents =  $540 \pm 10$  g;

$m_1$  = mass of the first empty cylinder =  $72 \pm 1$  g;

$M_2$  = mass of the second cylinder and its contents =  $940 \pm 20$  g;

$m_2$  = mass of the second empty cylinder =  $97 \pm 1$  g.

It is necessary to determine the total mass of chemical reagents, calculate the error of the total mass and record the final result.

5. Use the error calculation rules that are used when measuring independent measured values in the case of:

The specialist received the following measurement results:

$a=5 \pm 1$  cm;  $in=18 \pm 2$  cm;  $c=12 \pm 1$  cm;  $t=3.0 \pm 0.5$  s;  $m=18 \pm 1$  g.

Using the error rules of the sum (difference) of the measurement results and the product and division of the measurement results, calculate the errors and relative errors (in %):

a)  $a+v+c$ ; b)  $a+v-c$ ; c)  $c*t$ ; d)  $4a$ ; e)  $b/2$  (where numbers 4 and 2 have no error), f)  $m*b/t$ .

6. Use the calculation rules: errors of the sum and difference of values, errors of the product and division of the measurement results, the product of the measurement result by an exact number when calculating the following expressions:

a)  $(5 \pm 1) + (8 \pm 2) - (10 \pm 4)$ ; b)  $(5 \pm 1) * (8 \pm 2)$ ;

c)  $(10 \pm 1) / (20 \pm 2)$ ; d)  $2\pi * (10 \pm 1)$  (the numbers  $\pi$  and 2 have no error).

7. Use the rule for calculating the error when raising the measured value to the power in the case when the experimenter determines the acceleration of free fall  $g$  by measuring the time  $t$  of the stone falling from a height  $h$  ( $h$  is determined by the well-known formula  $h = \frac{1}{2}g * t^2$ ).

After several time measurements, he finds:

$t = 1.6 \pm 0.1$  s and measures the height  $h$  as  $h = 14.1 \pm 0.1$  m.

8. Use the error estimation formula using the satisfactory function of one variable in the case: the angle  $\vartheta$  was measured as  $125 \pm 2$  degrees. This value is then used to calculate  $\text{Sin}(\vartheta)$ .

$\text{Sin}(\vartheta)$  and error must be calculated.

9 Use the general formula for calculating errors in indirect measurements and the step-by-step method on the example of calculating a certain value  $c=a*v$ :

$a=10.0 \pm 0.5$  N;  $in=15 \pm 1$  cm.

10. The experimenter measures the density of the liquid five times and gets the results (in g/cm<sup>3</sup>): 1.8; 2.0; 2.0; 1.9; 1.8.

What can be said about the best estimate and margin of error based on his measurements?

11. It is necessary to accurately measure the area ( $S$ ) of a rectangular paper sample intended for testing with a size of 2.5 cm \* 5.0 cm.

In the table the results of 10 measurements of the width ( $l$ ) and length ( $c$ ) of the sample are given.

Dimension size	Measured values
<b>L</b>	24,25; 24,26; 24,22; 24,28; 24,24; 24,25; 24,22; 24,26; 24,23; 24,24.
<b>B</b>	50.36; 50.35; 50.41; 50.37; 50.36; 50.32; 50.39; 50.38; 50.36; 50.38.



In order to measure the area ( $S$ ) of a rectangular sample of paper, it is necessary, accordingly, to calculate the average value of both values, the standard deviation  $\sigma_x$  and standard deviation of the mean  $\sigma_{\bar{x}}$  using appropriate formulas.

12. Calculate the errors of the difference of measurement results in order to compare two measured values and use it for the case:

In an experiment to check the law of conservation of momentum, the student obtained the values given in the table for the initial and final moments ( $L$  and  $L'$ ).

Do you need to add additional columns for the difference ( $L - L'$ ) and error to the table and check whether the student's results are consistent with the law of conservation of angular momentum?

Primary ( $L$ )	Final $L'$
3.0±0.3	2.7±0.6
7.4±0.5	8.0±1.0
14.3±1.0	16.5±1.0
25±2	24±2
32±2	31±2
37±2	41±2

13. Determine (by filling in the free columns in the table) whether the sample of observations on the growth of 200 aborigines on some island corresponds to a normal Gaussian distribution, given in the table:

Bin number	Growth in bin	The number of observations Ok in the bin	The expected number of $E_k$	Pk, %
1	less than $X - 1.5\sigma$	14		
2	between $X - 1.5\sigma$ and $X - \sigma$	29		
3	between $X - \sigma$ and $X - 0.5\sigma$	30		
4	between $X - 0.5\sigma$ and $X$	27		
5	between $X$ and $X + 0.5\sigma$	28		
6	between $X + 0.5\sigma$ and $X + \sigma$	31		
7	between $X + \sigma$ and $X + 1.5\sigma$	28		
8	greater than $X + 1.5\sigma$	13		

14. Calculate the confidence interval in the case when the experimenter repeatedly measures  $g$ , the acceleration of free fall, and gets a result of 9.5 m/s<sup>2</sup> and a standard deviation equal to 0.1.

If we assume that the results of his measurements are normally distributed with a center equal to the accepted value of 9.8 m/s<sup>2</sup> and a width of 0.1, then what is the probability of obtaining a result that differs from 9.8 m/s<sup>2</sup> as much (or more) as the experimenter's result?

Assuming that the experimenter did not actually make mistakes, could it be said that his experiment was probably affected by some undetected bias?

15. Two measurements of the destructive force  $P$  give the results: 334±1 and 336±2.

Can these two results be considered consistent?

If so, then you need to calculate the best estimate of  $P$  and its error.

16. To determine whether it is necessary to reject a questionable measurement result in the case of: The specialist measures the thickness of the cardboard  $H$  ten times and gets the results (in mm): 0.86; 0.83; 0.87; 0.84; 0.82; 0.95; 0.83; 0.85; 0.89; 0.88.

a) It is necessary to calculate the average value  $\bar{H}$  and standard deviation  $\sigma_H$  of these results.

b) If the specialist decides to use the Chauvin test, should he reject the result of 0.95 mm? It is necessary to argue the concession.

17. Use formulas for calculating the average value and standard deviation of the average for the case:

The specialist measures the value of  $x$  five times and gets the results: 5, 7, 9, 7, 8.

It is necessary to calculate  $\bar{x}$  and standard deviation  $\sigma_x$ . Compare two options (with  $N$  and  $N-1$ ) during the calculation  $\sigma_x$ .

18. Use the error estimation formula for the use of a satisfactory function of one variable in cases of indirect measurements: the angle  $\vartheta$  was measured as  $\theta = 20 \pm 3$  degrees. This value is then used to calculate  $\text{Cos } \theta$ .

It is necessary to calculate  $\text{Cos } \theta$  and error.

19. To determine whether it is necessary to reject a questionable measurement result in the case of: The specialist makes 14 measurements of the oscillation period of the generator and receives the results (in fractions of a second): 7, 3, 9, 3, 6, 9, 8, 7, 8, 12, 5, 9, 9, 3

Feeling that the result (12) is too large, the specialist decides to use the Chauvenet criterion. Will he reject this result? How many results, similarly different from the mean as 12, should he expect?

20. Use the error estimation formula for the use of a satisfactory function of one variable in cases of indirect measurements: the angle  $\vartheta$  was measured as  $\theta = 20 \pm 3$  degrees. This value is then used to calculate  $\text{Cos } \theta$ .

It is necessary to calculate  $\text{Cos } \theta$  and error.

21. Determine the value of the  $\chi^2$  criterion for a sample of 40 measurements  $x_1, x_2, \dots, x_{40}$  of the length of the trajectory  $x$  of a bullet leaving a gun (the results are shown in the table).

731	772	771	681	722	688	653	757	733	742
739	780	709	679	760	748	672	687	766	645
678	748	689	810	805	778	764	753	709	675
698	770	754	830	725	710	738	638	787	712

22. Calculate the average value and standard deviation of the results of ten measurements of one of the indicators characterizing paper quality (for example, paper smoothness): 86, 85, 84, 89, 86, 88, 88, 85, 83, 85.

23. Calculate the confidence interval in the case when the experimenter wants to check the law of conservation of energy for a certain nuclear reaction. For this purpose, he conducts an experiment and obtains the results of the initial and final energy, respectively,  $E_p = 75 \pm 3$  MeV and  $E_q = 60 \pm 9$  MeV, where the standard deviations of the results are given as errors.

Is this difference significant (at the 5% level)? It is necessary to give a reasoned answer to the question.

### **Working program of the academic discipline (syllabus):**

**Compiled** associate professor, Ph.D., Ploskonos V.G.

**Approved** department \_\_\_E and TRP\_\_\_ (protocol No. 14 dated 18.05.2023)

**Agreed** Methodical commission of the IHF (protocol No. 10 dated 05/26/2023)